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Introduction

A Fabry-Perot etalon is an optical interferometer in which a beam of light undergoes multiple reflections between two reflecting surfaces, and whose resulting optical transmission (or reflection) is periodic in wavelength. The simplest etalons are uncoated plane-parallel solid etalons where the optical transmission and reflection are determined completely by the length of the etalon and its index of refraction. Fabry-Perot etalons can be used as precise wavelength references in telecommunication applications where the periodicity of the signal provides an array of reference frequencies for the telecommunication frequency grid. When used with active feedback electronics, Fabry-Perot etalons form the basis of wavelength-locking systems that can be used to stabilize the wavelength of a laser.

Theory of Ideal Fabry-Perot Etalons

Introduction: the plane-parallel etalon

The plane-parallel etalon acts as a frequency filter, or interferometer, through the interaction of multiple reflections from the partially reflecting dielectric interfaces of the etalon. The etalon performs a simple transfer function of changing optical frequency into transmitted intensity. As an example, consider an uncoated solid etalon made of fused silica in an ambient medium of air. Figure 1 shows the multiple reflections that the input beam undergoes as it traverses the etalon. In this example, light enters an etalon of length l from the left with incident angle θ_0 .

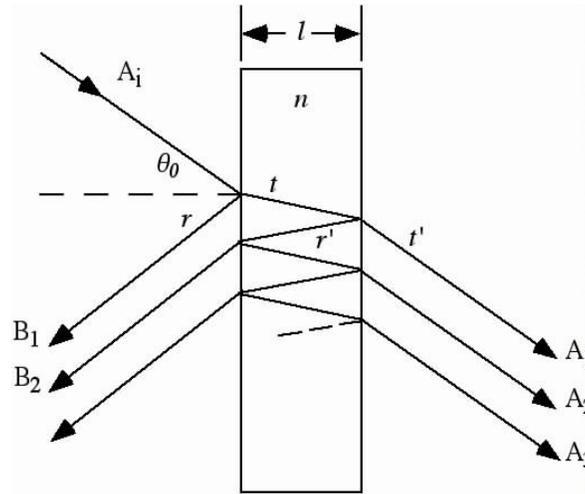


Figure 1 - Fabry-Perot Etalon: Multiple Reflection Model.

Starting with the amplitude of the incident electric field, A_i , the reflected amplitude from the first interface is given by B_1 , while the partially transmitted amplitude from the second interface is given by A_1 . The coefficients of amplitude reflection, r , and transmission, t , denote light traveling from air to silica while the coefficients, r' and t' , denote light traveling from silica to air. The multiple output beams differ in phase due to the different path lengths traversed by each of the beams. The optical phase acquired by the light on one round trip through the etalon is given by:

$$\delta = \frac{4\pi n l \cos \theta}{\lambda} \quad \begin{array}{l} n = \text{the index of refraction} \\ \text{where the } l = \text{thickness of the etalon} \\ \lambda = \text{wavelength of the laser} \end{array} \quad \text{Eq. 1}$$

The amplitudes of each of the transmitted waves can thus written:

$$A_1 = tt' A_i, A_2 = tt' r'^2 e^{i\delta} A_i, A_3 = tt' r'^4 e^{2i\delta} A_i, \dots \quad \text{Eq. 2}$$

The sum of transmitted wave amplitudes, A_t , is:

$$A_t = A_i tt' (1 + r'^2 e^{i\delta} + r'^4 e^{2i\delta} + \dots) = \frac{tt'}{1 - rr' e^{i\delta}} A_i. \quad \text{Eq. 3}$$

The fractional output intensity, or power transmission, $T = I_t / I_i$, from the ideal etalon is given by:

$$T = \frac{I_t}{I_i} = \frac{A_t A_t^*}{A_i A_i^*} = \frac{(tt')^2}{(1 - rr')^2 + 4\sqrt{rr'} \sin^2(\delta/2)}. \quad \text{Eq. 4}$$

In a lossless system, and with $r = r'$ for identical etalon surfaces, this equation simplifies to

$$T = \frac{1}{1 + F \sin^2(\delta/2)} \quad \text{with } F = \frac{4R}{(1 - R)^2},$$

where we have introduced the power reflectivity $R = r^2$, with lossless interface $r^2 + t^2 = 1$. This function is known as the **Airy function**, shown in Figure 2.

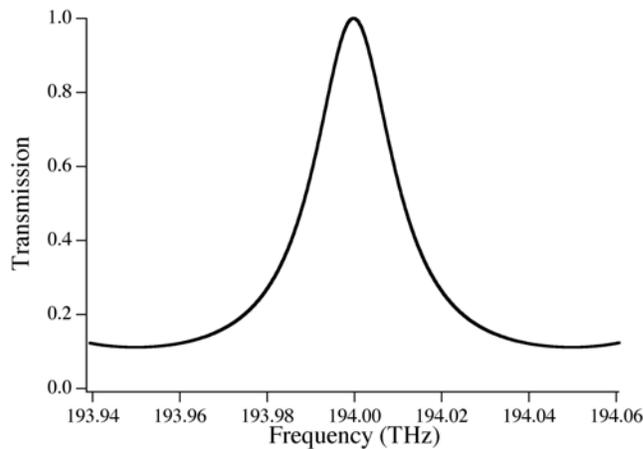


Figure 2 - Airy Function (Transmitted Intensity versus Frequency)

Important parameters of the plane-parallel etalon

Several important parameters describe an etalon: the optical wavelengths of maximum transmission, the *free spectral range (FSR)*, and the *finesse*. The wavelengths of maximum transmission occur periodically, and the spacing between adjacent maxima is called the FSR. Since the spacing is constant when expressed in frequency units, the natural unit of measure for etalons is frequency rather than wavelength. Units of frequency will be used for the remainder of this technical note. Finally, the finesse describes the narrowness of the peaks relative to the spacing between the peaks.

To locate the peaks in Equation 4 we use Equation 1 and find that the transmission of the etalon is a maximum when the phase difference for a round-trip pass is:

$$\delta = \frac{4\pi n l \cos \theta}{\lambda_m} = 2m\pi \text{ where } m = \text{any integer.}$$

Expressing the maximum condition in terms of frequency gives a more useful expression for the location of transmission peak locations:

$$\nu_m = m \frac{c}{2nl \cos \theta} \text{ where again } m = \text{any integer.} \quad \text{Eq. 4}$$

With the frequencies of the maximum transmission peaks known, the frequency separation between successive peaks can be calculated. The peak-to-peak separation in frequency is known as the **free spectral range (FSR)**:

$$\text{FSR} = \Delta \nu \equiv \nu_{m+1} - \nu_m = \frac{c}{2nl \cos \theta}. \quad \text{Eq. 5}$$

At normal incidence, the frequency spacing between maximum transmission peaks in an ideal etalon depends inversely on the index of refraction, n , times the thickness, l . The product of these two values, $n \cdot l$, is known as the **optical path length (OPL)**.

While the OPL determines the peak locations for a given frequency, the reflectivity of the etalon determines the shape of the peaks. The narrower the peak required, the higher the necessary reflectivity. The **finesse, F** , is commonly used to quantify the shape of the etalon transfer function. Finesse is defined as:

$$F \equiv \frac{\pi \sqrt{R}}{1 - R} \text{ where } R = \text{the reflectivity of the etalon.} \quad \text{Eq. 6}$$

A finesse of $F = 100$ requires a reflectivity of 97% along with excellent surface finish. A more common reflectivity range in the telecommunications industry is 25-50%. Figure 3 shows the theoretical transmission functions of etalons with differing reflectivity. The finesse of the resonant cavity is related to the quality factor (Q-factor).

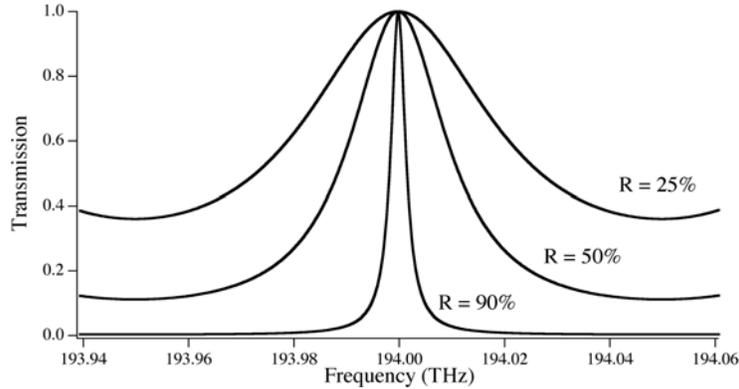


Figure 3 - Etalon Transmission: Finesse at Different Reflectivities, (FSR = 100 GHz)

Interdependence of free spectral range and peak location

While the choice of material and the thickness used for an etalon gives a wide range of design options, the free spectral range (FSR) and the location of peaks of maximum transmission cannot both be chosen arbitrarily. The equations above show the frequencies of maximum transmission are just integer multiples of the FSR for an uncoated, dispersion-free etalon.

For example, a hypothetical dispersion-free glass with an index of 1.444 and an FSR of 50.00 GHz would have a thickness of 2.076 mm. The frequencies of maximum transmission are multiples of the FSR, putting maximum transmission peaks in the center of the C-band at 194.000 THz (1545.5 nm.), 194.050 THz, (1545.1 nm), 194.100 THz (1544.7 nm), etc. Real glasses, such as fused silica, are not so kind, having a wavelength dependent index, but the link between the transmission maxima and the FSR remains.

Summary

This article summarizes the elementary theory of Fabry-Perot etalons. It highlights the periodicity of the optical transmission and its use as a stable frequency reference in telecommunications. The addition of optical coatings enhances the transmission characteristics showing how it may be used as a simple optical filter. In the following article, the realities of etalon construction will be discussed, including the tradeoffs such as materials, temperature sensitivities, and surface finish.